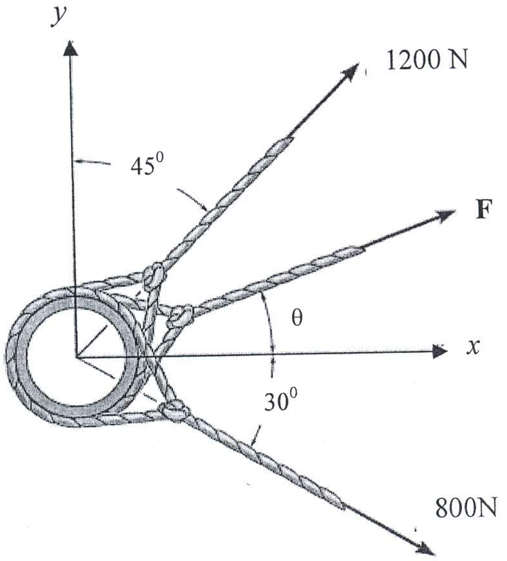
Problem I:



Determine the angle **θ** so that the magnitude of force **F** in this cable is minimum, if:

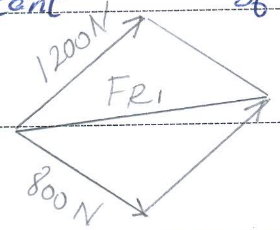
a- The three cables create a resultant force F_R having a magnitude of 1800 N

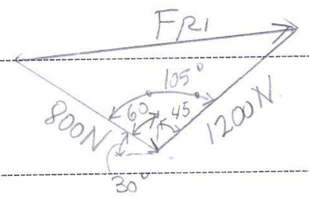
b- The resultant force F_R is directed along a line measure 10^o clockwise from the positive x axis. *(20 points)*

Calculations:

a- the parallelogram Law of addition and the triangular rule are shown:

1- Finding the resultant of the two known forces. F_{R1}





By applying the law of cosines:

$$F_{R_1} = \sqrt{800^2 + 1200^2 - 2(800)(1200)\cos 105} = 1605.2\ N$$

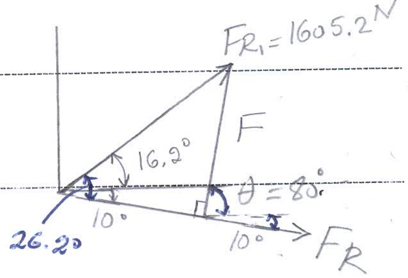
Applying the law of sines

$$\frac{1605.2}{\sin 105} = \frac{1200}{\sin \theta} \Rightarrow \theta = 46.2^\circ$$

when F is directed along F_{R_1}, F will be minimum to create the resultant force

$\Rightarrow F = 1800 - 1605.2 = 194.8\,N$ $\xrightarrow{F_{R_1}}$ $\xrightarrow[F_R]{}$ $F = F_{min}$ and $\theta = 46.2^\circ$

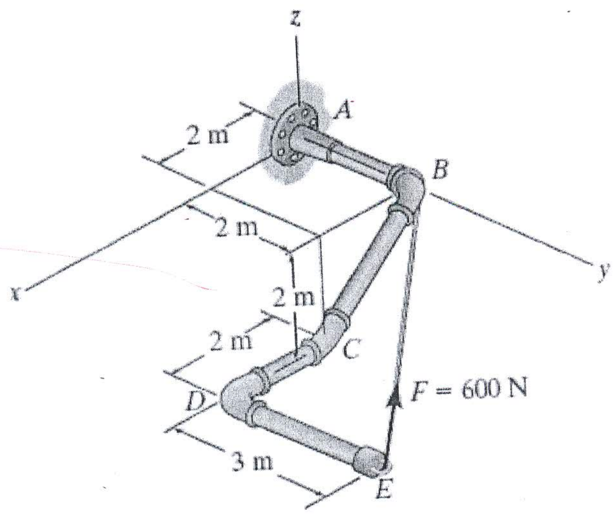
b- For F to be minimum, it has to be directed perpendicular to F_R



$$\therefore \sin 26.2^\circ = \frac{F}{1605.2} \Rightarrow F = 709.2\,N$$

and $\theta = 80^\circ$

Problem II:



Determine the magnitudes of the components of F=600 N acting along and perpendicular to segment DE of the pipe assembly. *(15 points)*

Calculations:

the unit vectors $\vec{u}_{ED}$ and $\vec{u}_{EB}$ must be determined.

$B(0, 2, 0)$ $D(4, 2, -2)$ $E(4, 5, -2)$

$$\vec{u}_{EB} = \frac{\vec{r}_{EB}}{r_{EB}} = \frac{(0-4)i + (2-5)j + (0+2)k}{\sqrt{(-4)^2 + (-3)^2 + (2)^2}} = -0.743i - 0.557j + 0.37K$$

$$\vec{u}_{ED} = \frac{\vec{r}_{ED}}{r_{ED}} = \frac{(4-4)i + (2-5)j + (-2+2)k}{\sqrt{(0)^2 + (+3)^2 + (0)^2}} = \frac{-3j}{\sqrt{(3)^2}} = -j$$

thus, the force vector $\vec{F}_{EB}$ is given by:

$$\vec{F}_{EB} = F\vec{u}_{EB} = 600\{-0.743i - 0.557j + 0.37k\} \{-445.8i - 334.2j + 222k\} N$$

∘ Vector Dot product: the magnitude of the component F parallel to segment DE of the pipe assembly is:

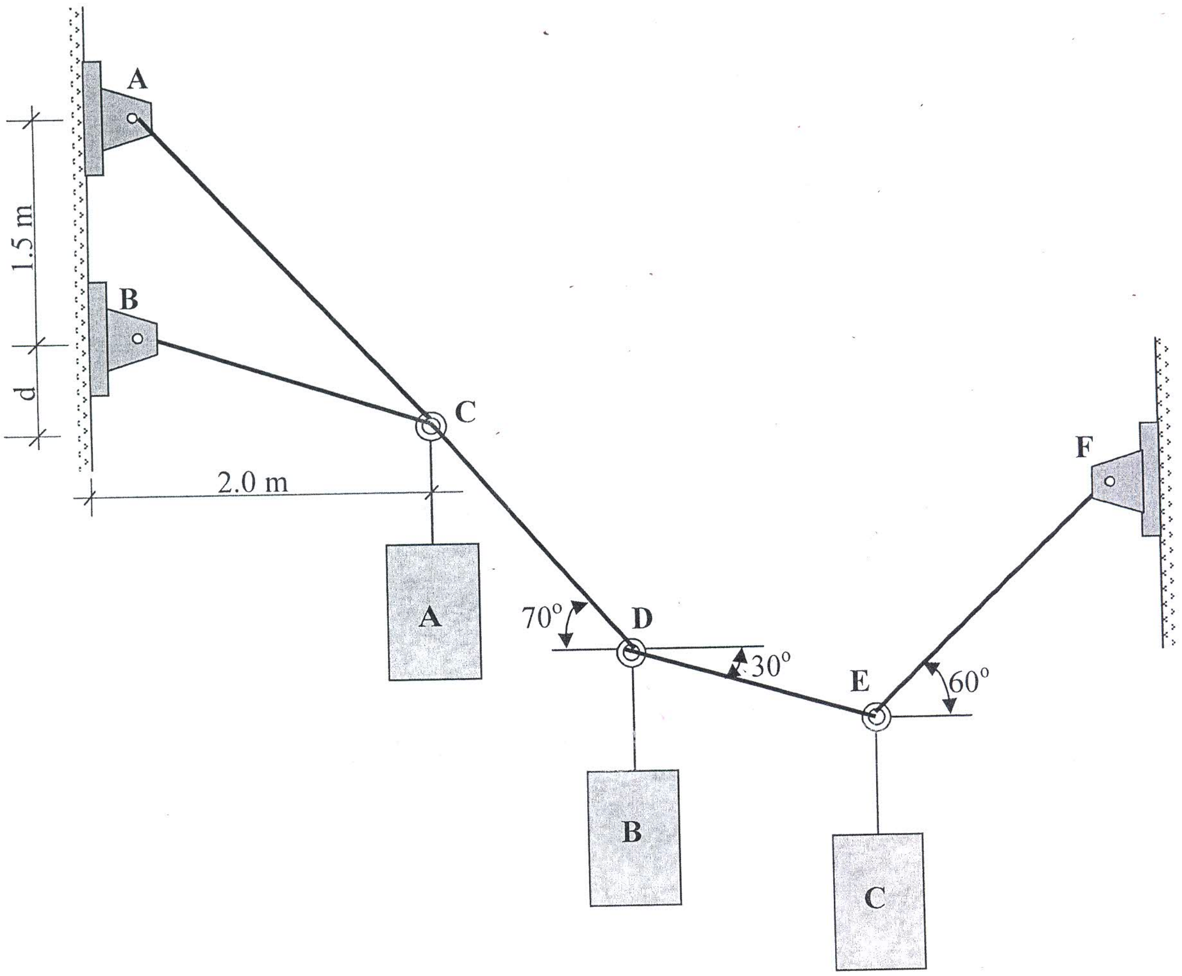
$(F_{ED})_{//} = \vec{F}\,\vec{u}_{ED} = -(-445.8i - 334.2j + 222k)(-j)$

$= 334.2\ N$

∴ the component of F ⊥ to segment DE of the pipe assembly is:

$(F_{ED})_{\perp} = \sqrt{F^2 - (F_{ED})_{//}^2} = \sqrt{(600)^2 - (334.2)^2} = 498\ N$

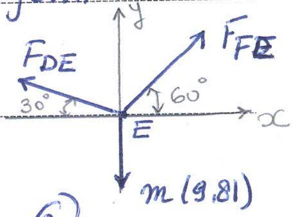
Problem III:



If the bucket *A* weighs 1.5 kg and the bucket *C* weighs 2 kg. Determine the required weight of the bucket *B* and the dimension *d* so that the force in cable *BC* is zero for equilibrium. *(30 points)*

Calculations:

Equations of Equilibrium: Apply the equation of Equilibrium along x and y axes to the free body diagram of joint E:

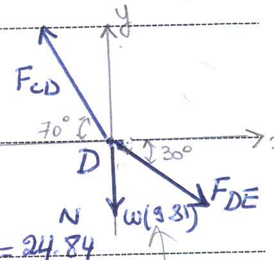


$\xrightarrow{+} \Sigma F_x = 0$ $F_{FE} \cos 60 - F_{DE} \cos 30 = 0 \Rightarrow F_{FE} = 1.732 F_{DE}$ ---- (1)

$+\uparrow \Sigma F_y = 0$ $F_{FE} \sin 60 + F_{DE} \sin 30 - 2(9.81) = 0$ ---- (2)

Substitute eq. (1) in (2) $\Rightarrow$ $F_{DE} = 9.81\,N$ and $F_{FE} = 16.99\,N$.

* Using the result of $F_{DE} = 9.81\,N$ and applying the equilibrium along x and y axes to the free-body diagram of joint D:



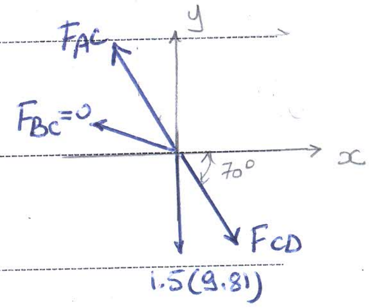
$\overset{+}{\rightarrow}\sum F_x = 0 \Rightarrow +F_{DE}\cos 30 - F_{CD}\cos 70 = 0$

$\Rightarrow +9.81\cos 30 - F_{CD}\cos 70 = 0 \Rightarrow F_{CD} = 24.84\,N$

$+\uparrow\sum F_y = 0 \Rightarrow -F_{DE}\sin 30 + F_{CD}\sin 70 - w(9.81) = 0 \Rightarrow -9.81\sin 30 + 24.84\sin 70 = w(9.81)$

$\Rightarrow w = 1.88\,Kg.$

* Using the result of $F_{CD} = 24.84\,N$ and applying the equilibrium along x and y axes to the free-body diagram of joint C.



$\overset{+}{\rightarrow}\sum F_x = 0 \quad F_{CD}\cos 70 - F_{AC}\left(\dfrac{2}{\sqrt{2^2+(d+1.5)^2}}\right) = 0$

$\Rightarrow \dfrac{F_{AC}}{\sqrt{2^2+(d+1.5)^2}} = \dfrac{F_{CD}\cos 70}{2} = \dfrac{24.84\cos 70}{2} = 4.25\,N$

$+\uparrow\sum F_y = 0 \Rightarrow -F_{CD}\sin 70 + \underbrace{F_{AC}\dfrac{d+1.5}{\sqrt{2^2+(d+1.5)^2}}}_{4.25} - 1.5(9.81) = 0$

$\Rightarrow -24.84\sin 70 + 4.25(d+1.5) = 1.5(9.81) \Rightarrow d = 7.46\,m.$

$\therefore F_{AC} = 4.25\left(\sqrt{2^2+(d+1.5)^2}\right) = 4.25\sqrt{2^2+(7.46+1.5)^2}$

$\Rightarrow F_{AC} = 39.02\,N$